technical monograph 8

Generalized Control Theory

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Introduction

The purpose of this manual is to introduce the basic ideas of automatic control theory. It is intended either as a primer for those embarking on a rigorous study of control theory or to provide a background for those seeking only a general knowledge of the analytical approach to control dynamics.

Section I – Block Diagram Techniques

Block Diagram Construction

One of the very powerful tools used in system analysis is the block diagram. It provides an exceedingly simple method of pinning down the exact responsibility of each element involved in a given system. For our purposes any combination of elements, however they may be grouped, that fall within any prescribed set of boundaries is defined as a “system”.

To illustrate the construction of a block diagram, assume that the behavior of any element in a system can be described by the ratio of its output to its input. For example, a diaphragm actuator has an input of diaphragm pressure, (P), and an output of stem movement, (Y). The actuator would then be described by the ratio (Y/P). For another illustration, the input to a valve is stem motion, (CY), and its output is flow, (W). The valve can now be described by the ratio (W/Y). With the help of one more ratio, a simple system can be described. Consider a level “process”. The input is flow, (W), and the output is level or head, (h). The describing ratio for the process is (h/W). Combining the elements, a system such as shown in Figure 1-1 might result.

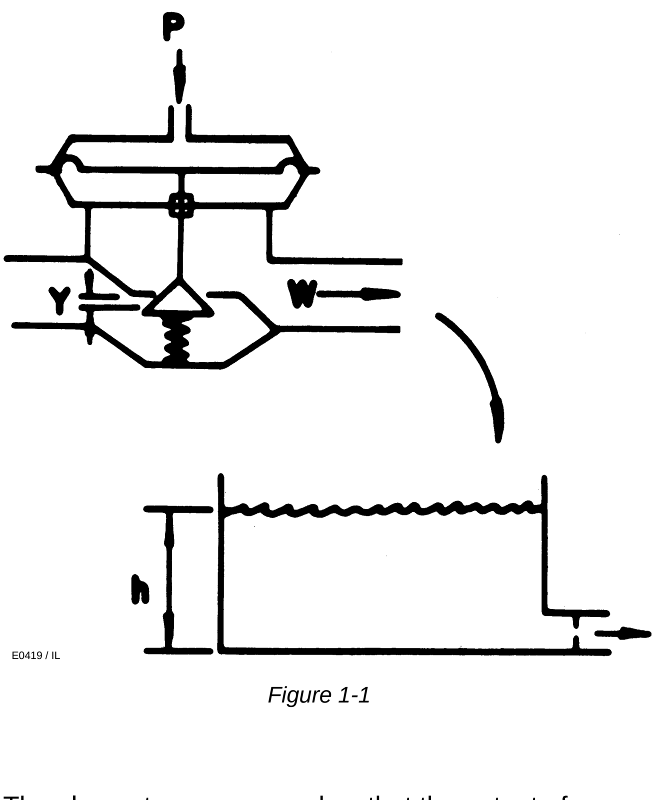


Figure 1-1

The elements are arranged so that the output of one becomes the input of the next. In Figure 1-2 each element has been replaced by a block. The function of each block is represented by the describing ratio of the element it replaces.

The block diagram should be thought of as an information system rather than a duplicate of the physical system it describes. The inputs and outputs of a block diagram consist of information about some variable in the system and do not necessarily represent the addition or removal of energy or mass.

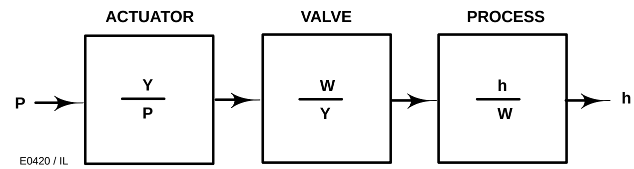


Figure 1-2

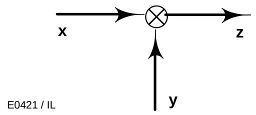
The diagram indicates that if the actuator block is multiplied by (P), the output is (Y), and (Y) times the valve block produces (W), and (W) times the process block produces the final output (h). In other words, the solution of each block is superimposed, in a mathematical sense, on the next succeeding block.

No information about the final output is returned to the input in this arrangement, i.e., there is no feedback in this system. Any such arrangement is called an “open loop” control system. All open loop control systems have the following characteristics in common:

1. Under constant load (outflow from the tank in the example) the output can be controlled by the input only if each element in the system is carefully calibrated.
2. This system is completely incapable of compensating for changes in load or changes in calibration due to part wear.
3. This system is inherently stable; there is no possibility whatsoever for this system to cycle.

Because the first two characteristics of the open-loop system seriously limit its control accuracy, it might be desirable to add some form of feedback. That is, in some way allow information about the output variable to be communicated back to the input and exert a corrective influence. A common method of introducing feedback in a system such as the example would be to include a proportional controller. It would measure the level, (h), and compare it with a reference value or setpoint, (r). The controller would produce a diaphragm pressure (P), proportional to the difference between (h) and (r).

To incorporate this into the block diagram, one more notation is needed, the summing point. This is represented simply by:



which means that x + y = z. The quantities (x) and (y) are added algebraically to cover the usual case of a difference in sign. The new system is shown in Figure 1–3. Note that, within the controller, the error, (e), equals (r – h) although the describing ratio for the controller as a unit is (P/h).

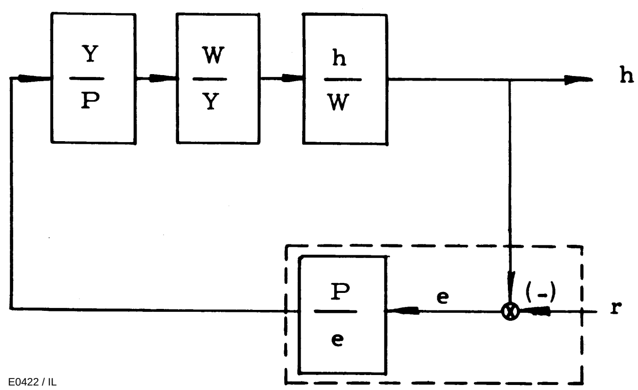


Figure 1-3

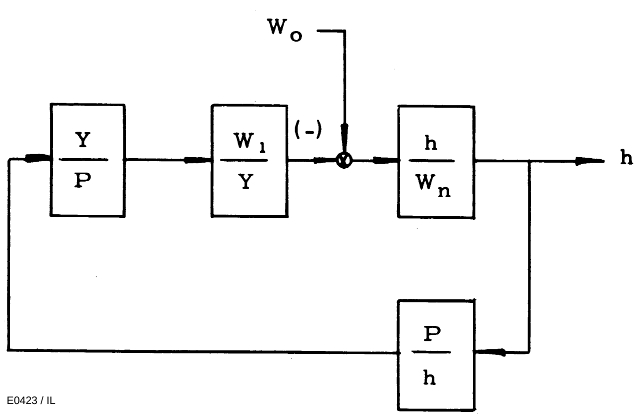


Figure 1-4

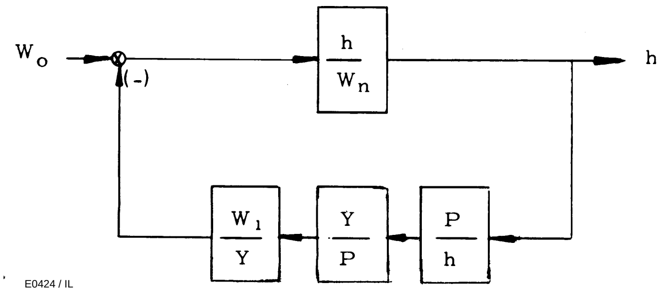


Figure 1-5

The only input shown is the set point or reference, (r). The output is the level, (h). The servo-mechanism engineer would be concerned with how the output variable, (h), varies following changes in the reference value, (r). In other words, he is concerned with a "following problem".

In process control, the value (r) is usually held constant and the input variable is really the change in load. The control engineer is interested in how the output, (h), varies with a change in load or he is concerned with a "load problem". It is perfectly legitimate to use information about the load as an input to the "information system" or block diagram even though the load represents the removal of mass from the physical system.

To show how load changes effect the system, only one more summing point need be included. Also, it should be made clear that the level, (h), depends on the net flow, (W_n), which is the difference between the inflow through the valve (W_1), and the load or outflow, (W_o).

$$W_n = W_1 - W_o$$

Figure 1–4 shows the block diagram with a load input. The diagram becomes a little more clear as a load problem if it is rotated counter clock- wise slightly as shown in Figure 1–5.

The negative sign at the summing point merely indicates that the output of the valve has the opposite effect of the load. This is an example of a negative feedback control loop often termed a "closed-loop" control system. Closed-loop control systems have the following characteristics:

1. Compensation for load changes and element wear will occur in accordance with the elements in the feedback path.

2. Depending on the combination of elements in the loop, an unstable or oscillating system may exist.

These two statements really say that the introduction of feedback may improve the quality of control, but the price paid for improvement is that the problem of stability will have to be dealt with.

Block Diagram Algebra

In the preceding examples each block represented the output-input ratio for some element in the system. The output of each element was obtained by multiplying the describing ratio by an input. Using this principle and by superimposing successive solutions, a feedback control system was described. The input to the control system was the load, (W_0), and the system output was level, (h). To see how the output varies with the input, it is necessary to determine the describing ratio for the entire closed-loop system, (h/W_0). For simplicity, assume that the ratio in the forward path is (G) and that the product of the ratios in the feedback path is (H). For the level control example then:

$$G = \frac{h}{W_n} \text{ and } H = (P/h)(Y/P)(W_i/Y) = W_1/h$$

The diagram for this example is shown in Figure 1-6.

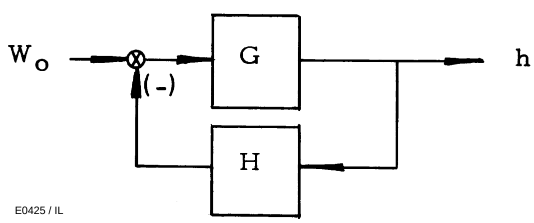


Figure 1-6

The output of the feedback path must be the product of the input to the feedback, (h), and the feedback block, (H) or (hH). The output of the forward path must be the product of the input to (G) and (G) itself. The input to (G) is the algebraic sum of the negative feedback and the load or:

$$\text{input to } G = W_O - hH$$

The output from (G) must be:

$$G(W_O - hH) = GW_O - GhH$$

and the output must equal (h), therefore:

$$GW_O - GhH = h$$

or:

$$GW_O = h + GhH$$
$$= h(1 + GH)$$

$$\frac{h}{W_o} = \frac{G}{(1 + GH)}$$

The behavior of the closed–loop system with negative feedback can now be represented by one block, see Figure 1-7.

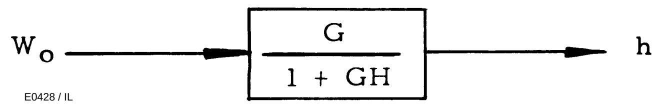


Figure 1-7

To illustrate how more complex systems can be simplified with this one relationship, two examples are given in Figures 1–8 and 1-9.

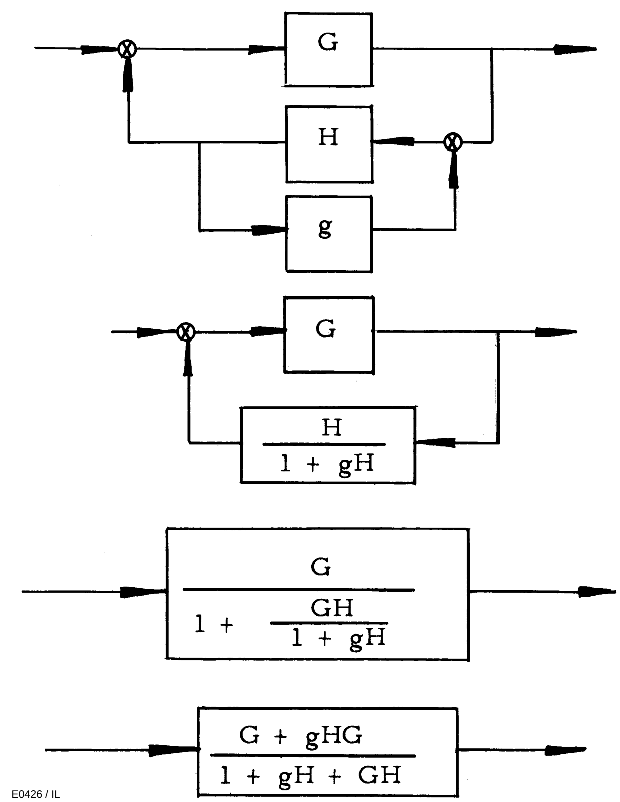


Figure 1-8

Notice that in every case the most subordinate loop is closed first and that series elements are multiplied when there are no branches either entering or leaving between them.

The analysis of any control system, regardless of how complex or elementary, should begin with a block diagram. All information concerning transient response (stability) and steady–state response can be determined by operations performed on the block diagram and on information gained from it.

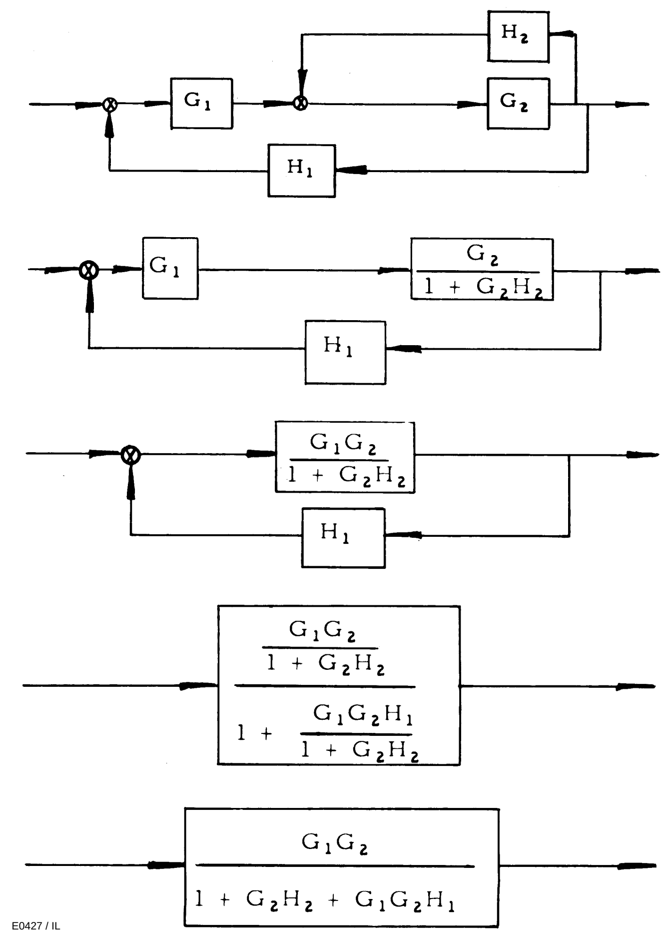


Figure 1-9

Mathematical Descriptions

Thus far it has been assumed that the behavior of any element can be described by the ratio of its output to its input. Within certain limitations, this is possible. When working with linear elements (a linear element will be defined later), the describing ratios that have been used would be in the form of a "transfer function". A transfer function is, by definition, the ratio of the Laplace transform of the output to the Laplace transform of the input.

Obtaining a transfer function for an equation is normally a very simple operation. In general, each differential is replaced by the Laplace operator, (s), a second differential, by (s^2), and so on. An integral is replaced by (1/s). A variable such a (x) is written $\mathcal{L}(x)$ to indicate the Laplace transform of (x). Table 1 shows some examples.

Table 1

The Laplace transform of this –	is this
x	$\mathcal{L}(x)$
$\frac{dx}{dt}$	$s\,\mathcal{L}(x)$
$\frac{d^2x}{dt^2}$	$s^2\,\mathcal{L}(x)$
$\frac{d^3x}{dt^3}$	$s^3\,\mathcal{L}(x)$
$\int x dt$	$1/s\,\mathcal{L}(x)$
$\frac{dx}{dt} + x = y$	$s\mathcal{L}(x) + \mathcal{L}(x) = \mathcal{L}(y)$

Table 2

Element	Typical describing equation(1)	Laplace transformed equation	Transfer function
controller	$P = Kh$	$\mathcal{L}(P) = K\mathcal{L}(h)$	$\mathcal{L}(P)/\mathcal{L}(h) = K$
actuator	$T/a\ dy/dt + 1/a\ y = P$	$T/a\ s\mathcal{L}(y) + 1/a\ \mathcal{L}(y) = \mathcal{L}(P)$	$\mathcal{L}(y)/\mathcal{L}(P) = a/(T_S + 1)$
valve	$W_1 = NY$	$\mathcal{L}(W_1) = N\mathcal{L}(y)$	$\mathcal{L}(W_1)/\mathcal{L}(y) = N$
process	$W_n = c\ dh/dt$	$\mathcal{L}(W_n) = Cs\mathcal{L}(h)$	$\mathcal{L}(h)/\mathcal{L}(W) = 1/C_S$

1. These equations are merely presented as typical and do not necessarily refer to previously discussed equations.

The transformed equation can be manipulated like any algebraic equation. Differentiation can be performed merely by multiplying the transformed equation by (s), integration by (1/s). Notice that before transformation the equation is a function of time, (t), but after transformation it is a function of the Laplace operator, (s). Returning the equation to a function of time is called finding the "inverse function". If a number of operations have been performed on the transformed equation, finding the inverse function may be a difficult task. To help with this, sets of tables are available for most of the common forms of transfer functions. The entire procedure becomes somewhat analogous to the use of logarithms. Logarithms are added and subtracted to accomplish the operations of multiplication and division. The real solution is then the anti-log, most easily obtained from a set of tables. Laplace transformed equations are multiplied and divided by the Laplace operator, (s), to accomplish the operations of differentiation and integration. The real solution is then the inverse function, most easily obtained from a set of tables.

The procedure presented here is entirely mechanical and no mathematical proof is given. The nature of the Laplace operator itself will not be discussed other than to say that it can be treated like any algebraic variable. It should be pointed out for the benefit of those intending further study of Laplace transforms that the technique presented here does not yield the complete

Laplace transform equation. However, all quantities that have been neglected are initial conditions, and it is always possible in control systems to assume a datum plane that renders the initial conditions zero.

The utility of the Laplace transform lies in the fact that the output-input ratio or transfer function can be written for an equation containing differential quantities as well as for algebraic relationships. This permits the use of block diagrams and the superposition principle outlined previously.

To illustrate the use of transfer functions, the describing ratios in the level control example will be defined. Figure 1–10 shows how the block diagram would be written in transfer function form. The value of each transfer function is given in the last column of Table 2.

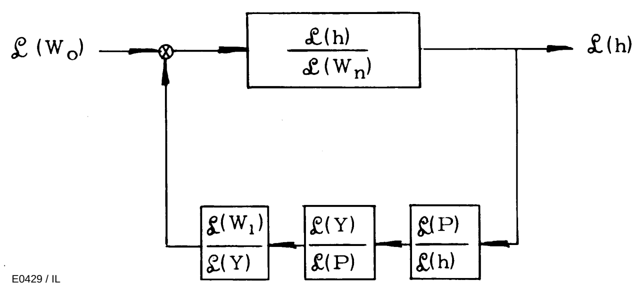


Figure 1-10

As was inferred earlier, the Laplace transform exists only for linear elements. A linear element is one whose behavior can be described by a linear algebraic or differential equation with constant coefficients. That is, an equation containing no products or differential products of the dependent variable and whose coefficients do not vary.

In Table 3, equation No. 4 is non–linear because (x^2) is a product of (x) and itself. In equation No. 5, the second quantity is the product of the (x) variable and its differential. In equation No. 6, (y) appears to the minus one power.

Table 3

Linear Equations	**Non–linear Equations**
(1) $ax + b = y$	(4) $ax^2 + b = y$
(2) $\frac{(d^2x)}{dt^2} + \frac{dx}{dt} + ax = y$	(5) $\frac{(d^2x)}{dt^2} + x\frac{dx}{dt} + bx = y$
(3) $\frac{(d^2x)}{dt^2} = a\frac{dy}{dt} + by + c$	(6) $\frac{dx}{dt} = a\frac{dy}{dt} + \frac{b}{y} + c$

As serious as this limitation first appears, a great many physical elements that are of interest in control can be described by linear equations. Those that can not usually can be considered linear for small excursions about some operating point. By investigating a number of operating points throughout the range of performance of a system, a good overall picture can be obtained. The use of “linear theory” in this way has found widespread and successful application in the servo-mechanisms and communications fields.

Section II – Oscillation and Stability

The topic under discussion in this section is system stability. Working from the block diagram techniques developed in Section I, a method of explaining the principle of oscillation and determining stability is presented. The problem of indicating relative stability is also discussed.

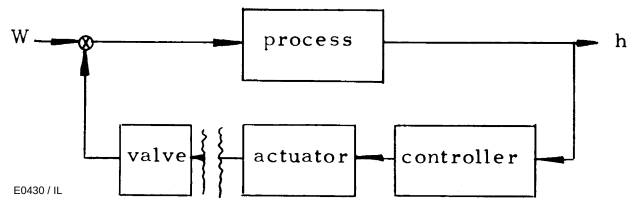


Figure 2-1

Frequency Response Method

In studying the dynamic behavior of a given control element, it is logical to supply the element with a dynamic or changing input. A comparison of the output with the input would reveal the dynamic behavior of the element. A satisfactory analysis could be made with any type of varying input. It has evolved, however, that the interpretation of the results becomes much easier and more effective if the input is restricted to a sine-wave of appropriate amplitude and if the response of the element is observed as only the frequency is varied. This technique is aptly called "frequency response".

If a linear element is supplied with a sufficiently slow sine-wave input, the output will be a sine-wave of the same frequency and will occur in phase (simultaneously) with the input[1]. The output amplitude may be greater or less than the input and may differ dimensionally. The ratio of the output amplitude to the input amplitude is called "gain". During very low frequency inputs when there is no phase shift, the gain is commonly referred to as the "zero frequency gain" or static gain". As the frequency of the input is increased, two things will occur: (1) the phase between the output and the input will shift with the output usually lagging, (2) the gain of the element will change with the output usually being attenuated. If the gain at any frequency is divided by the static gain, the resulting quantity is called the "magnitude ratio". Knowledge of the static gain and of the phase shift and magnitude ratio throughout an appropriate frequency spectrum will completely describe the dynamic behavior of the element. When a series of linear elements are connected output to input and the system is subjected to the frequency response technique of analysis, the overall phase shift is the sum of the individual phase shifts, the overall magnitude ratio is the product of the individual magnitude ratios, and the overall static gain is the product of the individual static gains.

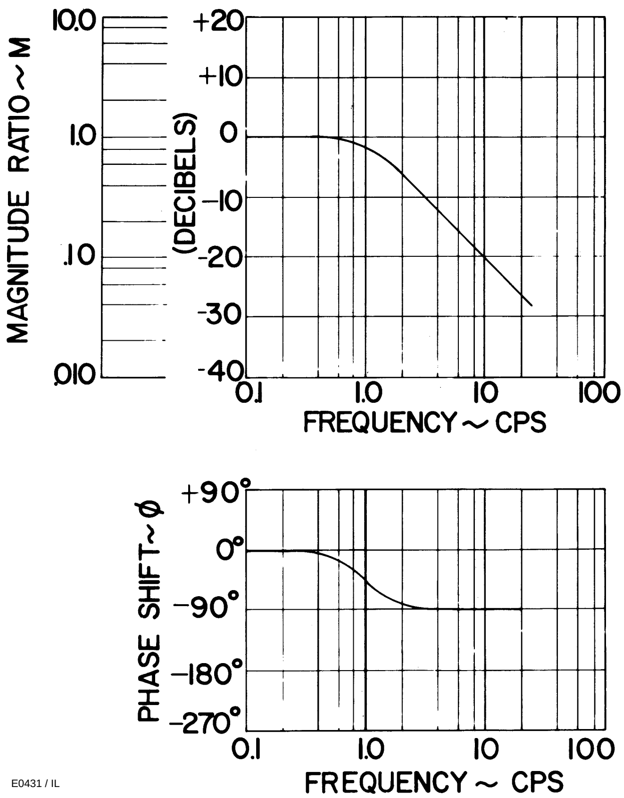
How the quantities of phase shift and gain vary for each element in a feedback or closed- loop system will determine the stability of that loop. For an illustration of the conditions of stability, refer to the level control example in Section I and the diagram in Figure 2–1.

Suppose the chain of information is broken somewhere in the loop, say between the actuator and the valve stem as shown in Figure 2-1. Now study the open–loop system starting with the valve stem and ending with the actuator stem and including all the elements of the closed-loop system. Assume that the valve stem is provided with a sine-wave input and observe the output at the actuator stem. The ratio of the amplitude of the actuator output to the amplitude of the valve input is called "open-loop gain". It will be the product of the individual gains of each element in the system. The open-loop phase shift will be the sum of all the individual phase shifts in the loop.

Generally, each element will contribute some phase lag or negative phase shift; and, since the feedback is negative, an additional 180^o lag will be acquired at the summing point. Consider, now, changing the frequency of the sine-wave input to the valve until the open-loop phase shift shows that the output is lagging the input by a full 360^o. The output will appear to be back in phase with the input. At this frequency, if the open-loop gain is one, the valve and actuator stems will be moving with the same amplitude. It follows that, if the stems were connected at any given instant, the output of the actuator would become the input of the valve. Since each output cycle is the result of one previous input cycle, the oscillations would be self-sustaining. This is the fundamental principle of oscillation. It can be shown that any linear closed-loop system that has an open-loop gain of one at 360^o phase lag will cycle continuously. Furthermore, it will oscillate at the exact frequency that produces 360^o phase lag. If the gain at 360^o phase lag is greater than one, the system will oscillate with ever increasing amplitude. This is because the amplitude at any point in the loop, at any instant, will equal the product of the open-loop gain and the amplitude of the previous cycle at that point. If, however, the open-loop gain is less than one at 360^o phase lag, the oscillations will die out.

A very important fact concerning the stability of linear systems is now evident: stability is determined by the open-loop gain at 360^o phase lag and is completely independent of load disturbances.

1. Technically, there are exceptions to this statement, but they may be safely ignored for the purposes of this discussion.



The magnitude ratio is either plotted on a logarithmic scale or the logarithm of the magnitude ratio is plotted on a linear scale. A common practice, and the one used here, is to convert the magnitude ratio, (M), into decibels or db.

db = 20 $\log_{10}$M

Figure 2-2

Since the feedback is almost always negative, it is conventional not to speak of the –180° phase shift at the summing point. Reference is normally made only to the –180° phase shift accumulated by the elements in the loop, and stability is then said to depend upon the open-loop gain at 180° phase lag.

Frequency response data can be determined experimentally or by mathematical analysis. The information is usually determined for individual elements and plotted in any of several ways. Two common methods of plotting frequency response data are the Bode plot and the Nyquist diagram. They are illustrated in Figures 2-2 and 2-3 respectively. The Bode plot is on rectangular coordinates. The phase shift and the log of the magnitude ratio are both plotted as functions of the log of the frequency. The Nyquist diagram is a polar plot. The vector of the plot has a length equal to the magnitude ratio and an angle equal to the phase shift. While these plots are normally made for individual elements, the values of all the elements in a control loop can be combined on one plot to determine the stability of that loop.

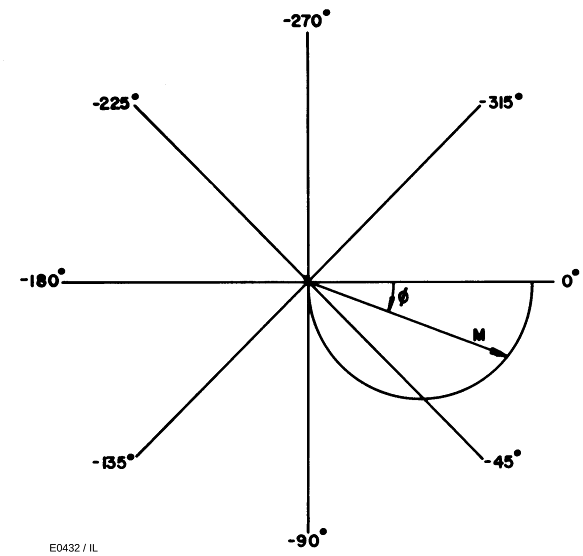


Figure 2-3

Relative Stablility

We have shown that the stability of a system depends upon the open–loop gain and the open–loop phase shift as functions of frequency. We have also stated that the transfer function of an element completely describes its dynamic behavior. Therefore, there must be some relationship between frequency response and transfer functions. This relationship will be shown without presenting the mathematical basis for it.

Assume that some element is described by the first order differential equation:

$$\frac{dy}{dt} + ay = bx$$

Think of (x) as the input and (y) as the output of the element. This equation is linear and, if the coefficient (a) and (b) are constant, it can be Laplace transformed.

$$\frac{dy}{dt} + ay = bx$$

$$\mathcal{L}(y)\,s + \mathcal{L}(y)a = \mathcal{L}(x)b$$

$$\mathcal{L}(y)\,(s + a) = \mathcal{L}(x)b$$

The ratio of the Laplace transforms of output to input is the transfer function of the element.

$$\frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} = \frac{\mathcal{L}(y)}{\mathcal{L}(x)} = \frac{b}{s + a}$$

Divide numerator and denominator of the right hand side by (a):

$$\frac{\mathcal{L}(y)}{\mathcal{L}(x)} = \frac{b/a}{(1/a)\ s + 1}$$

If we now define two new symbols so that:

$G_0 = (b/a)$ and $T = 1/a$

We can write the transfer function as:

$$\frac{\mathcal{L}(y)}{\mathcal{L}(x)} = G_o \ \frac{1}{T_s + 1}$$

The quantity (G_0) is a function of the two constants, (a) and (b), and so is itself a constant. It is, in fact, the static gain or zero frequency gain of the element. The quantity, (T), depends only on the constant, (a), and so is also a constant. Because of its location next to the operator, (s), it is called a “time constant”. The entire quantity [1/(Ts + 1)] involves a differential term, (s), and is therefore time dependent. The frequency response of this element is shown in Figure 2-4. The frequency response curves are really a plot of the time dependent part of the transfer functions, that is, [1/(Ts + 1)] versus frequency. The time constant, (T), is related to the frequency at which the magnitude ratio falls off, as shown in Figure 2-4. The phase is shown in the figure to lag as frequency increases. Because of this laging characteristic and because the defining equation is first order, the element is called a “first order lag”.

A second order differential equation of common occurrence is:

$$m \ \frac{d^2y}{dt^2} + b \ \frac{dy}{dt} + ky = x$$

The transfer function for the equation is:

$$\mathcal{L}(y)\ s^2m + \mathcal{L}(y)sb + \mathcal{L}(y)k = \mathcal{L}(x)$$

$$\mathcal{L}(y)\ (s^2m + sb + k = \mathcal{L}(x)$$

$$\frac{\mathcal{L}(y)}{\mathcal{L}(x)} = \frac{1}{ms^2 + bs + k} = (1/k)\frac{1}{\frac{m}{k}s^2 + \frac{b}{k}s + 1}$$

If we define the following new terms:

$$G_o = 1/k, \ \ T = \sqrt{m/k}, \ \text{ and } \zeta = (b/2)\sqrt{1/km}$$

We can then write the transfer function as:

$$\frac{\mathcal{L}(y)}{\mathcal{L}(x)} = G_o\frac{1}{T^2s^2 + 2T\zeta s + 1}$$

Again, the term (G_0) is the static gain and is a constant. The quantity (T) is the time constant and is related to the magnitude ratio curve as shown in Figure 2-5. The term (ζ) is called the “damping ratio”.

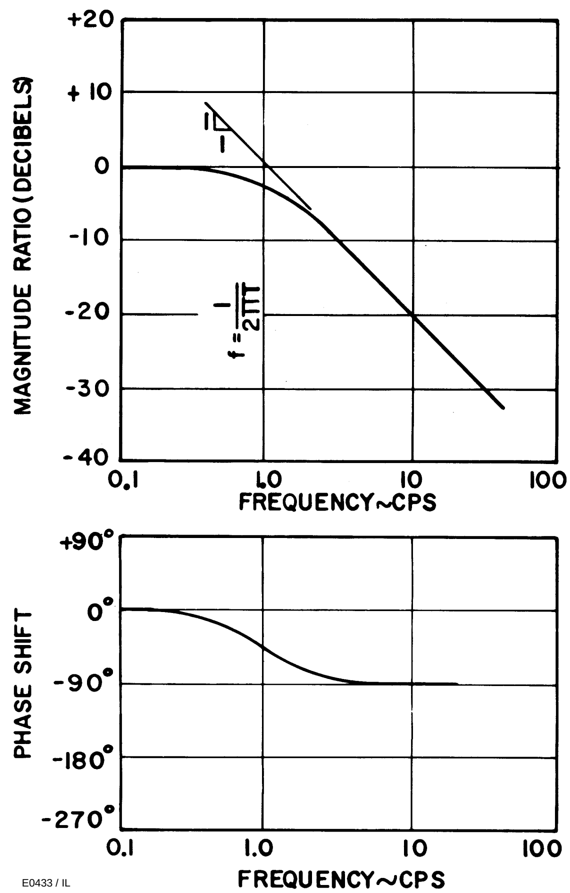


Figure 2-4

Its effect on the magnitude ratio is also shown in Figure 2-5. Notice that the magnitude ratio curve eventually becomes asymptotic to a line whose slope is (–1) for a first order lag and (–2) for a second order lag. This rule can be extended to any order system. A second rule is that the maximum phase shift for a first order element is 90°, for a second order element 180°, and so on.

To summarize, we can say that both the transfer function and the frequency response curves state:

$$Actual \ \ gain = \frac{Output}{Input} =$$

(static gain) (magnitude ration), at an angle ∅

or in symbols:

$G = (G_0)\ (M)\ \angle\varnothing$

where:

G	= actual gain at any frequency
G_0	= static or zero frequency gain
M	= magnitude ratio
∅	= phase shift between output and input

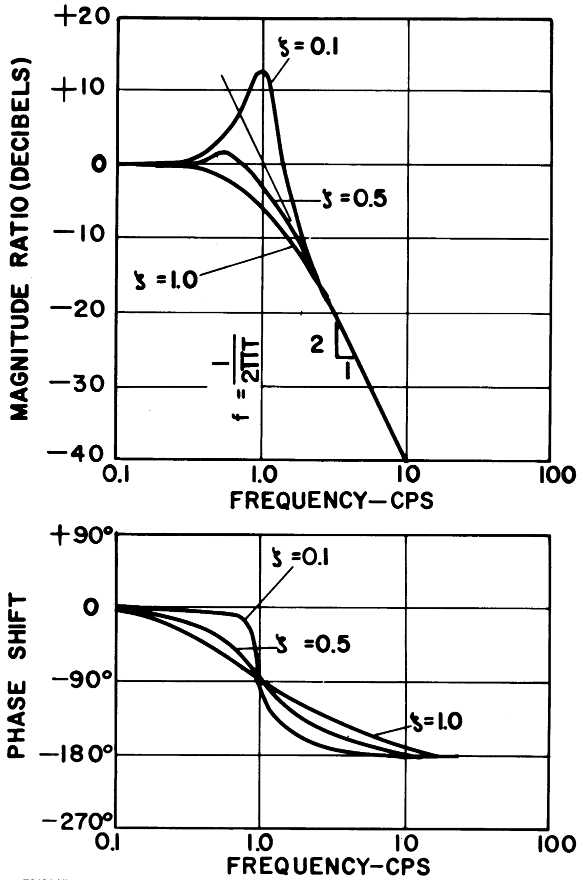


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Figure 2-5

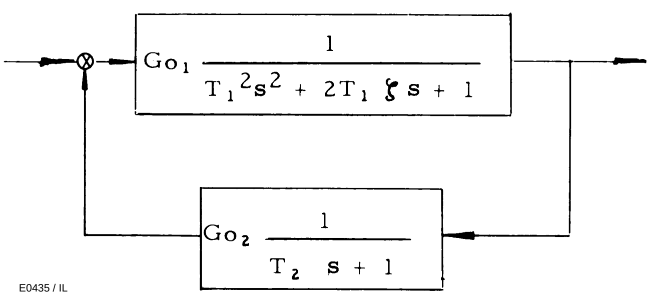

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Figure 2-6

To illustrate the use of frequency response in determining stability, consider the system in Figure 2-6. The frequency response curves for this system are both plotted and labeled in Figure 2–7. The open–loop transfer function is the product of the two individual transfer functions:

open–loop transfer function =

$$G_{o1} \frac{1}{T_1^2 s^2 + 2T_1 \zeta s + 1} \quad G_{o2} \frac{1}{T_2 s + 1}$$

The total open-loop phase shift is the sum of the individual phase shifts and is shown in Figure 2–7. The open–loop gain is the product of both static gains and both magnitude ratios.

$$\text{open-loop gain} = G = (G_{01}G_{02})(M_1M_2)$$

For convenience the magnitude ratio is converted to decibels which is a logarithmic scale. Therefore, the magnitude ratios (M_1) and (M_2) in Figure 2–7 can be multiplied simply by adding the decibel readings.

The stability criteria states that the gain must be less than unity at 180° phase lag. That is:

$$G = (G_{01}G_{Q2}M_1M_2) < 1, \text{ when } \varnothing 1 + \varnothing 2 = -180^o$$

The magnitude ratio curve shows an open-loop value of –15 db at the frequency which produces –180° phase shift.

$$M_1\,M_2 = -15 \text{ db} = 0.178$$

For a stable loop, then:

$$(G_{o1}G_{o2}) = \frac{1}{M_1\;M_2} = \frac{1}{0.178} = 5.61$$

In other words, if the product of the static gains, (G_{01}) and (G_{02}), is numerically greater than 5.61, the system is unstable.

We are now in a position to consider the concept of relative stability. Assume the example system with a static gain of 6 and oscillating violently. Now imagine that by turning a knob we can gradually decrease the gain. When the gain reaches a value slightly below 5.61, the system must be stable, but we would not expect it to suddenly snap into very solid, rock stable behavior. In fact, with gains just slightly below 5.61 the system will respond to disturbances with a great deal of oscillation, but the oscillations will always diminish and die out in time. As the gain is decreased further below the value 5.61, the system will get less and less oscillatory and eventually become extremely slow and sluggish. The question is now, “Relatively how stable should the system be for lively response with a minimum of oscillation?” There is no general answer to this question. If the open-loop transfer function is exactly known, an optimum value of gain can then be calculated, but the results will be different for each individual system. Some general rules can be shown to be quite helpful in many cases, however. Two common measures of relative stability are “gain margin” and “phase margin”. In a sense these measures provide a factor of safety on stability. Gain margin is the factor by which the open–loop gain must be multiplied to just reach the unstable condition. Thus in our example if the static gain were adjusted to equal 1.87, the gain margin would be calculated as follows:

$$\text{if } G_{01}\,G_{02} = 1.87, \text{ Gain margin } = \frac{5.61}{1.87} = 3$$

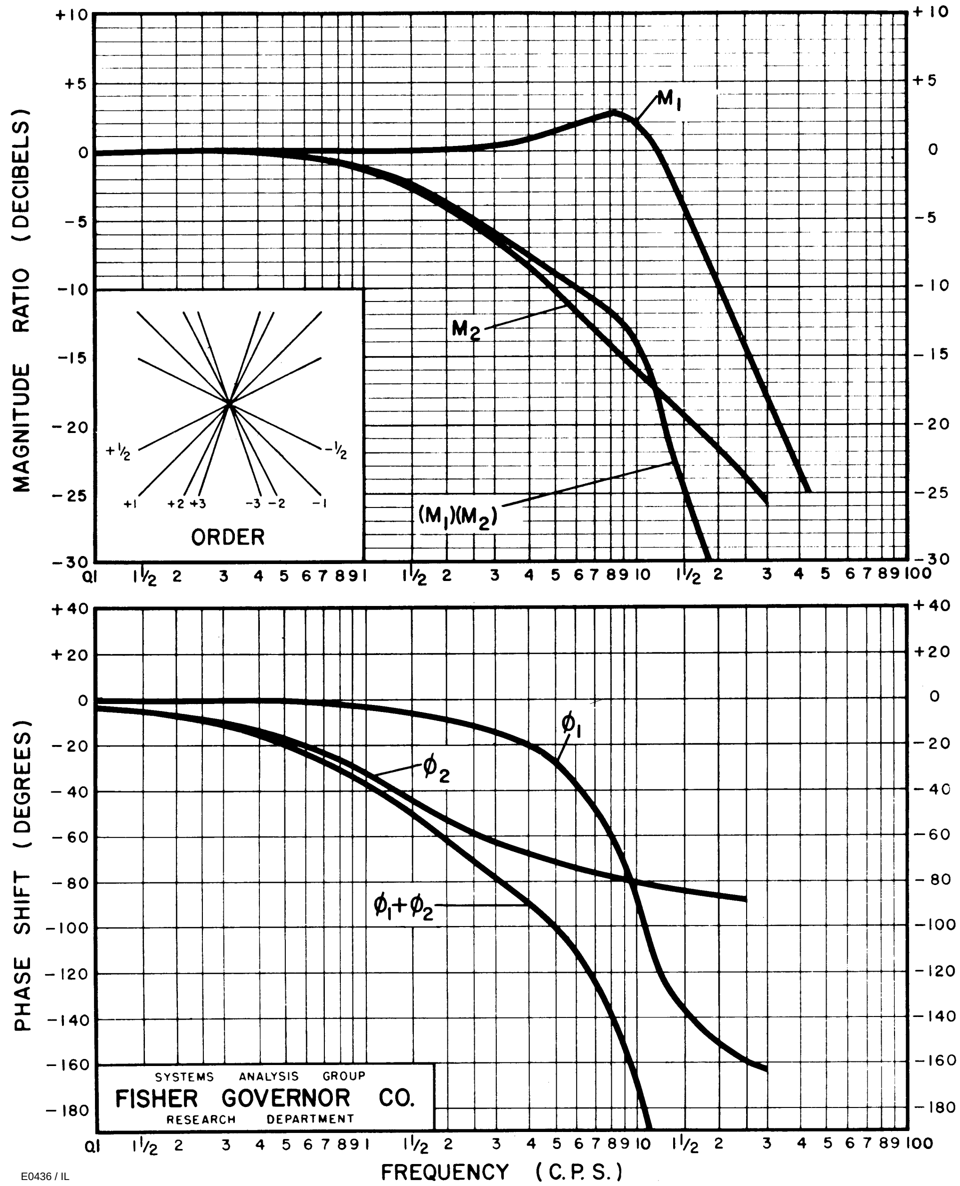

Figure 2-7

Phase margin is defined as the difference between 180º lag and the actual open–loop phase lag at the frequency where the open–loop gain is unity. If we want in our example a phase margin of 45º:

open-loop phase lag = 180º –45º = 135º lag

The frequency that gives 135º lag is shown in Figure 2–7 as 7.7 cps. The open-loop magnitude ratio is:

$$M_1\ M_2 = -11.4\ db = 0.27$$

The necessary static gain to provide 45º phase margin then is:

$$G_{o1}\ G_{o2}\ M_1\ M_2 = 1$$

$$G_{o1}G_{o2} = \frac{1}{M_1\ M_2} = \frac{1}{0.27} = 3.7$$

Therefore, if the static gain is adjusted to a value of 3.7, the system will have a phase margin of 45º. In this example the reader can verify that a phase margin of 45º will provide the same relative stability as a gain margin of (1.5). For low order systems, acceptable relative stability can be obtained with phase margins of about 45^{o} or gain margins around 2.

Other Techniques

The discussion in this manual has been restricted to the frequency response approach. Another significant approach in the analysis of linear or linearized systems is the “root locus” method. In this method, the roots of the system equation are plotted as the open–loop gain is varied, thereby obtaining a “locus of roots” as a function of gain. Interpretation of this plot then yields information about the system behavior.

A host of other techniques and concepts have been developed for dealing with both linear and non-linear systems. Techniques such as describing functions, transforms other than the Laplace transform, computer simulation, optomizing and adaptive control are all extremely important concepts in control theory. However, the author feels that the subjects introduced in this manual are in no way belittled by these more advanced techniques. Linear theory and the frequency response approach still has effective and widespread application and is an essential first step toward the more complex topics.

Section III – System Response

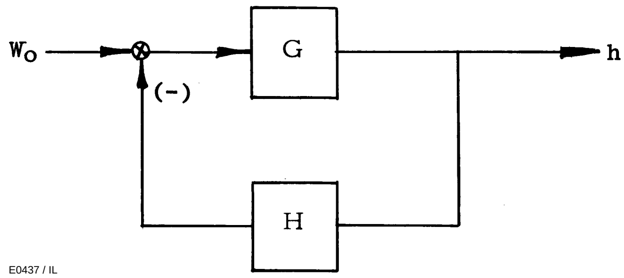

Figure 3-1

Section I presented the block diagram as a tool for describing a control system and for determining where relationships exist among the individual elements. Working from this foundation, Section II discussed the problem of oscillation which was shown to be the result of the open-loop parameters in a feedback loop. We shall now consider the relationship between input and output for the overall system. In other words, closed–loop performance.

Consider a single loop system with negative feedback as shown in Figure 3-1. The elements in the forward path are described by (G) and in the feedback path by (H). The input to the entire system is (W_O) and the output is (h). In Section I it was shown that the closed-loop transfer function for such a system is:

$$\frac{\mathcal{L}(h)}{\mathcal{L}(W_o)} = \frac{G}{1 + GH}$$

The output of the system is obtained merely by multiplying the transfer function by the input.

$$\mathcal{L}(h) = \mathcal{L}(W_o)\ \frac{G}{1 + GH}$$

A very important fundamental is now obvious; the output of a control system is a function of both the closed-loop transfer function and the input. This is contrasted to the stability which has been shown to be independent of the input.

A second important concept in closed–loop control arises when a system with high open–loop gain is considered. The reference here is to the static gain. In this case the product, (GH), will be large and the following approximation is very good:

$$\frac{\mathcal{L}(h)}{\mathcal{L}(W_o)} = \frac{G}{1 + GH} = \frac{G}{GH} = \frac{1}{H},\ \ where\ \ GH \gg 1$$

Thus, if the static open-loop gain is high, the closed-loop function reduces to the inverse of the feedback elements. The concept is obviously valuable where non-linear elements can be kept in the forward path. This also makes it possible to accurately vary the system response by manipulating only the feedback elements.

Conclusion

It has been the purpose of this manual to present the basic ideas of linear control theory in a generalized fashion. Control systems are dynamic in behavior. If the ever increasing requirements of control systems are to be met, it will be done by facing the dynamics problems involved. It is hoped that these few pages have done a little toward creating a understanding of the basic utility of control theory.

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